SCIENCE FOR GLASS PRODUCTION

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A METHOD FOR CALCULATION OF THE SHAPE OF BENT SHEET GLASS TRANSPORTED ON A ROLLER CONVEYER

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Based on the model developed by the Institute of Silicate Chemistry (ISC) of the Russian Academy of Sciences, a method is proposed for calculating the variations of geometrical parameters in bent sheet glass when transported on a roller conveyer. This method makes it possible to estimate theoretically not only the shape of the article after transportation, but also the process characteristics, based on the required deformation level.

Bent sheet glass is extensively used for glazing by contemporary architects; however, the industrial production of such glass involves numerous unresolved technical problems. One of the most significant problems is the changing of the shape of a product when being transported on a roller conveyer, which spontaneously occurs under the effect of gravity and residual internal stresses. This makes it necessary to predict the final shape of an article after transportation.

In developing a method to estimate variations in the shape of an article transported on a roller conveyor, we rely on the theory of deformation of plates and a model developed by the Institute of Silicate Chemistry which describes the physics of the glass deformation process [1, 2].

Let the molded sample represent a plate with curvature radius R directed along the axis x and having no initial curvature along the axis y. The sample can occupy two transitory positions on a single pair of rolls: *1 – 1* and *3 – 3* (Fig. 1), which correlate with the beginning and the end of the transportation process. In this case the maximum projections of the plate with respect to the transition roller will be equal to $L_x/2$, with the flight between the rollers equal to l_{br}.

The values of the bending moments will be determined according to the calculation scheme (Fig. 2) which represents a plate on two roller bearings with lengths of the projections L_1, L_2, and L_3. To simplify the calculation, let us take the beam deformation laws as the bending moment laws; since the plate thickness constitutes 1/2 of the plate length, the error will not exceed 5%.

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Then the bending moment values will be equal to:

on the segment $0 \le x \le L_1$:

$$M_{x_A} = -ql_y \frac{l_1 x_{d_1}}{2},$$

where q is the distributed load resulting from the plate gravity; l_y is the plate width; l_1 is the length of the projection of the first segment of the plate, calculated in accordance with the sample geometry; x_{d_1} is the distributed load arm on the first segment;

on the segment $L_1 \le x \le L_2$:

$$M_{x_B} = -ql_y l_1 (x_{d_1} + x) + R_A x - ql_y l_2 x_{d_2},$$

where x is the coordinate of the considered point; l_2 is the projection length of the second segment of the plate calcu-

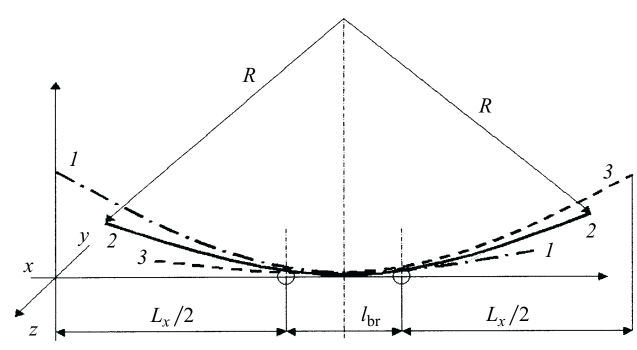

Fig. 1. Scheme of deformation of a glass plate with curvature radius R in transportation: *1 – 1*) the position of the article in the beginning of transportation; *2 – 2*) the same in the intermediate position; *3 – 3*) the same in the end of transportation.

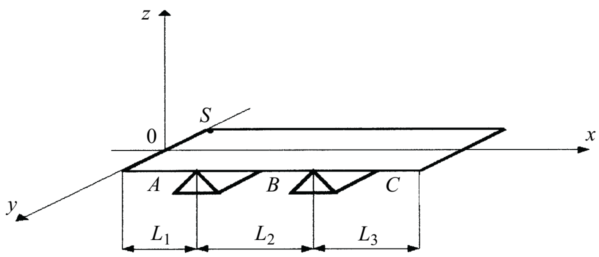

Fig. 2. Calculation scheme for determination of bending moments: A, B, C designate the plate projections.

lated in accordance with the sample geometry; x_{d_2} is the distributed load arm on the second segment; R_A is the reaction of the roller bearing;

on the segment $L_2 \le x \le L_3$

$$M_{x_C} = -ql_y\left(l_1(x_{d_1} + L_2 + x) - l_2\left(\frac{L_2}{2} + x\right) + l_3 x_{d_3} \right) + R_A(L_2 + x) - R_B x,$$

where R_B is the bearing reaction; l_3 is the projection length of the third segment of the plate calculated in accordance with its geometry; x_{d_3} is the distributed load arm on the third segment.

The reactions of the bearings R_A and R_B are calculated in accordance with the known laws of the strength of materials.

In the above calculations, the distributed load value is found from the formula

$$q = h\rho g,$$

where h is the plate thickness; ρ is the density of the sample material; g is the free fall acceleration.

The values of the bending moments along the axis y are determined in the same way.

To perform calculations based on the above formulae, it is necessary to determine the length of the plate projections between the rolls. As the article is moving, the values will vary with time.

Let the sample move at a constant linear speed v calculated from the formula

$$v = \frac{\pi n r}{30},$$

where r is the roll radius; n is the rotational speed of the rolls.

The speed of the sample motion is equal to the linear rotational speed of the outer surface of the rolls. The following boundary conditions have to be taken into account in solving this problem:

$$l_{1\max} = \frac{l_x}{2};\quad l_{1\min} = l_x - \frac{2\pi r}{180}\arcsin\left(\frac{l_{br}}{r}\right);$$

$$l_{3\max} = \frac{l_x}{2};\quad l_{3\min} = \frac{l_x}{2} - \frac{2\pi r}{180}\arcsin\left(\frac{l_{br}}{r}\right).$$

Variations in the plate projection length occur periodically, since the sample moving along the conveyor belt passes from one pair of rollers onto the next pair. The variation in the projection length during one cycle can be found from the following system of equations:

$$\left.\begin{aligned} l_{1j}(\tau) &= l_{1\max} - i\mathrm{d}\tau v; \\ l_{3j}(\tau) &= l_{3\min} + i\mathrm{d}\tau v, \end{aligned}\right\}$$

where 1, 3 are the numbers of the projections; i is the number of the pitch for the calculations within one cycle; $\mathrm{d}\tau$ is the calculation discreteness in time.

The determination of the values of the bending moments in each specific time moment is related to the need to determine the coordinates of the point of application of the force resulting from the distributed load.

In the considered case, when the article has no initial curvature directed along the axis y, such curvature can arise only on those segments in which the sample has no contact with the transporting rolls, whereas in the zone of contact with the roller, this particular curvature will be smoothed due to the motion on the conveyer belt. The secondary curvature value is calculated similarly to the calculation of the deformation along the axis x.

Let us determine the zone of contact between the article and the conveyer belt. In can be seen from Fig. 1 that the length of the contact of the moving sample with the conveyor constitutes $2l_{br}$ (one l_{br} on each side from the mass center along the axis x), i.e., deformation on the axis y in this segment is absent.

Let us calculate the sag values for each point at any time moment, based on the ISC model, and in doing so, let us determine the elongation of the bottom glass layer depending on the time and force parameters of the transportation process:

$$\left.\begin{aligned} \varepsilon_x &= -z\frac{\mathrm{d}^2\omega}{\mathrm{d}x^2}; \\ \varepsilon_y &= -z\frac{\mathrm{d}^2\omega}{\mathrm{d}y^2}, \end{aligned}\right\}$$

where ω is the sag for coordinate x; z is the coordinate in the sample bottom layer, for our case $z = h/2$.

Having twice integrated the above differential equations, we obtain the following system:

$$\left.\begin{aligned} \omega(x) &= \frac{\varepsilon_x}{h}x^2 + C_{1x}x + C_{2x}; \\ \omega(y) &= \frac{\varepsilon_y}{h}y^2 + C_{1y}y + C_{2y}. \end{aligned}\right\}$$

To solve this system, one should know the values of the coefficients C_{1x}, C_{2x}, C_{1y}, and C_{2y}. These values are deter-

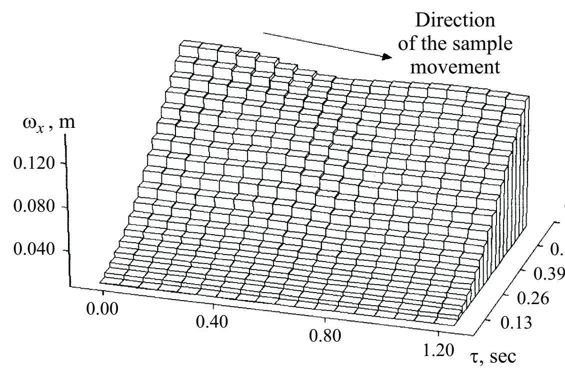

Fig. 3. Variations in the sag of the sample along axis x depending on the transportation duration: ω) sag along axis x; τ) transportation duration; l) half-length of the sample along axis x.

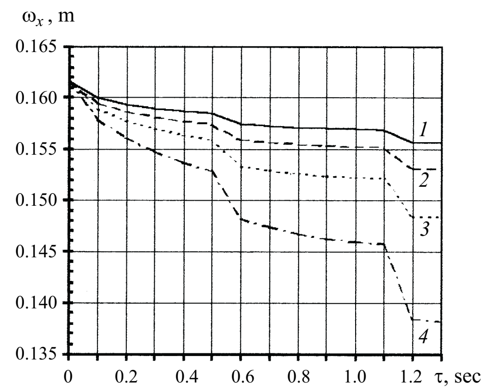

Fig. 4. Variation in the sag of point S (Fig. 2) along axis x at temperatures of the sample of 580 (*1*), 590 (*2*), 600 (*3*), and 610°C (*4*).

mined based on the boundary conditions:

$$\left.\begin{aligned} &\omega(y)=0, \quad y=0;\\ &\omega(x)=0, \quad x=L_1;\\ &\omega(x)=0, \quad x=L_1+L_2. \end{aligned}\right\}$$

For the specified conditions, the sought coefficients take the following values:

$$\left.\begin{aligned} &C_{1y}=0;\\ &C_{1y}=\pm\frac{2\varepsilon_y}{h}\frac{b}{2};\\ &C_{1x}=\frac{\frac{\varepsilon_x}{h}(L_1+L_2)^2-\frac{\varepsilon_x}{h}L_1^2}{L_1-(L_1+L_2)};\\ &C_{2x}=-\frac{\varepsilon_x}{h}L_1^2-C_{1x}L_1. \end{aligned}\right\}$$

Having determined the coefficient values, the sag values can be calculated as well. In this case, the elongation of the bottom fiber is calculated based on the formula of the ISC model, not taking into account the elastic component, which has the form:

$$\varepsilon(\tau)=\frac{\sigma}{\eta}\tau+\varepsilon_d(\infty)\left(1-\exp\left(-\left(\frac{\tau}{\tau_r}\right)^b\right)\right),$$

where σ is the stress acting inside the sample; τ is the process duration; η is the viscosity; τ_r is the relaxation coefficient; $b = 0.5$; ε_d (∞) is a coefficient determined according to the method described in [3].

The parameters for the ISC model are calculated taking into account the bending moments in each particular point of the plate. The calculation takes into account the total transportation duration, and not the duration of one cycle (that is, the movement over one pair of rolls).

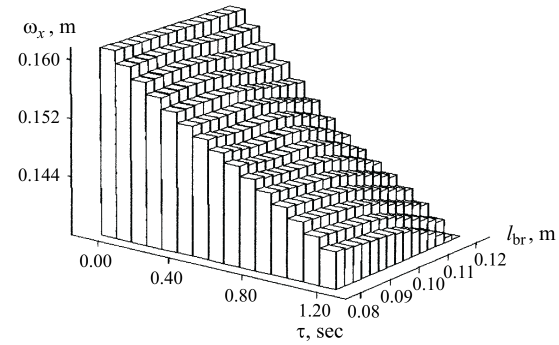

Fig. 5. Variations in the sample deformation at point S along axis x depending on the length between rolls (l_{br}) and transportation duration τ.

Since in most cases, the geometry of a bent sheet glass article is described by the values of the sags in respective coordinates, let us analyze the variations in this parameter in the course of transportation. The maximum sag will be equivalent to the sum of the minimum and maximum sags as the sample moves along the axis x.

Based on this method, the deformation of the article was calculated depending on the following factors: temperature t, transportation velocity v_{tr}, distance between the rolls $2l_{br}$. The calculation was carried out for the following initial data: $t = 580$, 590, 600, and 610°C; $v_{tr} = 0.2$ m/sec; $2l_{br} = 0.08$, 0.1, and 0.12 m. The following data were taken as constant ones for the process: transportation duration 1.2 sec, initial curvature radius 1.2 m, article length 1.2 m, article width 0.4 m, and thickness 0.0032 m, which corresponds to the actual article sizes.

Since the sample has a symmetrical shape, the plots (Figs. 3 – 5) are constructed for one quarter of the article, whereas the coordinate x is counted from the mass center of the sample (the article center). An assumption was made for the calculation that the residual stresses, which have not relaxed during exposure in the molding press, are minimal as compared to gravity and, therefore, can be neglected.

Let us construct plots based on the obtained results and analyze them.

The first plot (Fig. 3) shows the sag variation dependence for the transportation of a glass plate at temperature 610°C at the speed of 0.2 m/sec. The distance from the center of the article to the outer surface in the longitudinal direction is indicated on the axis *y*. Based on this plot, it is possible to predict the behavior of the article in any time moment and at any surface point, which makes it possible to account for this type of deformation in designing the molding equipment and selecting the technology parameters.

For a simpler description of the deformation process, most expedient is to consider sag variations at point *S* (Fig. 2). The deformation along the axis *x* for this point at different temperatures is described by the plots shown in Fig. 4. The presence of steps in the plots is accounted for by the variation of the length of projections, as the plate moves along the conveyer belt. The process temperature has a substantial effect on the level of deformation. As can be seen, the effect of the temperature on the deformation level is significant: the higher the temperature, the sooner the sample gets straightened.

To issue recommendations for modifying the technological parameters, it is necessary to identify the factors which have the greatest effect on the deformation of a sample transported on the roll conveyor. Let us consider the effect of the distance between the rolls for point *S* on the level of the sample deformation (Fig. 5).

Based on this plot, it can be stated that an increase in the distance between the rolls results in increasing sag values along the axis *x*, although the effect of this parameter is not so significant as that of the temperature,

Using the above data, it is possible to select expedient parameters for the distance between the conveyer rolls and for the process temperature, based on the prescribed sag values for the product; or else, knowing the technological process parameters, it is possible to determine the final shape of the product after transportation.

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